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ABSTRACT

This chapter discusses the way that mathematics curricula are developed in the United States. It suggests that curriculum development in mathematics education makes little progress because it fails to adhere to scientific methodology. Weaknesses in the current development process, how such development can become scientific, how curriculum development should be related to research on learning and teaching, and some possible criticism of taking a scientific approach are discussed. (KHR)

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25 Mathematics Curriculum Development as a Scientific Endeavor

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> If I have seen farther than others, it is because I have stood on the shoulders of giants.
>
> —Isaac Newton (as quoted by Steen, 1990b, p. 7)

Because of the chaotic way that mathematics curricula are developed in the United States, it is unlikely that we will soon hear Newton's words repeated by a curriculum developer in mathematics education. In this chapter, we argue that curriculum development in mathematics education makes little progress because it fails to adhere to scientific methodology. We discuss weaknesses in the current development process, how such development can become scientific, how curriculum development should be related to research on learning and teaching, and some possible criticisms of taking a scientific approach.

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A FLAWED PROCESS

A curriculum[1] is a detailed instructional blueprint for guiding students' acquisition of certain culturally valued concepts, procedures, intellectual dispositions, and ways of reasoning. Mathematics curricula link the practice of teaching to curriculum developers' knowledge and beliefs about mathematics, learning, teaching, schools, and culture. Ideally, mathematics curricula should form concrete ties that link research on learning and teaching to classroom instruction in a synergistic system.

But, belying its critical importance, the development of mathematics curricula in the United States is chaotic and suffers from major methodological flaws. First, almost anyone can engage in it. University professors and graduate students in mathematics and mathematics education, teachers, school administrators, parents, school districts, publishing companies, state departments of education, professional organizations, and funding agencies are all involved in the development of mathematics curricula. Curriculum development activities are taking place in National Science Foundation projects whose major goals are research, curriculum development, teacher enhancement, and systemic change. The difficulty with this plethora of players is that there are no established standards for development, peer review, communication, or professional training.

Second, too often the development of mathematics curricula is totally disconnected from research on students' mathematics learning. Frequently it is planned in "brainstorming" sessions in which the most prevalent justifications for curricular decisions are based on traditional "prerequisite analysis" ("We have to teach this because it is needed next year in..."), personal experience ("An activity that I used when I taught was . . ."), vague intuitions ("This doesn't feel right."), and duplication of previously published treatments ("Did you see the activity written by so and so?"). Such sessions rarely explicitly incorporate research findings and theoretical perspectives into instructional design. Some curriculum developers even state

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[1]Throughout this chapter, we use the term *curriculum* to refer to what many would call the "intended curriculum," as opposed to the "implemented curriculum."

privately that research on learning does not seem very helpful in designing curricula; so they make no attempt to carefully consider it during the design process. Additionally, curriculum materials relying on technology often are based more on what is possible with the technology rather than on what research suggests is possible or desirable for students (Battista, 1988).

Third, there is little reflection on or documentation of the *process* of developing mathematics curricula. Because this process has not been subjected to the critical scrutiny of scientific analysis and review, curriculum developers continually "reinvent the wheel," frequently evidencing little progress over previous efforts. In fact, K. J. Wilson and Daviss (1994) likened the current state of curriculum development in the United States to that of aeronautics before the time of the Wright brothers at Kitty Hawk:

> A century ago, people making airplanes were usually solitary, self-taught visionaries or eccentrics following their own theories or hunches. They lacked a good deal of information about aerodynamics. . . . They continued to work separately, often unknowingly crossing and recrossing each other's tracks, unable to take advantage of or build on each other's successes. (p. 98)

AN EXAMPLE: GENERAL GOALS, NEBULOUS OUTCOMES

The following draft of an instructional activity for second-grade students was created by a team of educators (including the authors) involved in an elementary mathematics curriculum development project funded by the National Science Foundation. The curriculum was intended to be consistent with the National Council of Teachers of Mathematics' *Curriculum and Evaluation Standards* (1989). Its general goals were: (a) to offer students meaningful mathematical problems; (b) to emphasize depth in mathematical thinking rather than exposure to a series of fragmented topics; and (c) to have students invent their own strategies and approaches rather than rely on memorized procedures.

In the following sections, we provide the lesson plan given to a teacher familiar with the goals of the curriculum. We then analyze the class discussion in which the teacher attempted to help students integrate the ideas they had worked on for several days.

The Lesson Plan

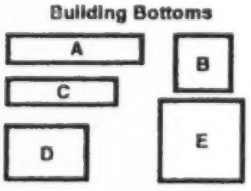
Goals. This lesson provides another context in which students can see several skip-counting sequences. It also will be good experience for students who in later grades are going to explore the enumeration of cubes in three-dimensional rectangular buildings.

Description. Students use interlocking cubes to construct buildings, pretending that each cube is a room in a building. The teacher shows three cubes attached together in a row (see FIG. 25.1) and says:

FIG. 25.1.

> This is a one-story building. How many rooms does it have? How many rooms will there be in the building if you make it two stories or floors high? There must be the same number of rooms on each floor. Try it and see. How many rooms would there be if the building were three stories high? Four stories? Can you predict, without building it, how many rooms would be in the building if it were 10 stories high?

The rectangles lettered A, B, C, D, and E show the outline of the bottom story of five different buildings (see FIG. 25.2). Students record in a table the number of rooms in each building if the building were one story high, two stories high, and so on.



Building	1 Story	2 Stories	3 Stories	4 Stories	5 Stories	10 Stories
A B C						
D E						

FIG. 25.2.

Suggestions for Class Discussion. Ask students to share their findings for Building A. As they state their results, record the numbers on the board: 5, 10, 15, 20, 25, 50. *Look at these numbers. Where have we seen this pattern before?* Students may point out that they are counting by fives or skip-counting. Ask them if they can explain why they are counting by fives.

Ask students to share results for Buildings B and C. *Why do the numbers come out the same?* Ask students to build three-story buildings for B and C and compare them. *What is the same about the buildings? What is different?*

Observation of the Final Class Discussion

- Teacher: Look at your numbers on the *Build a Building* sheet. Tell me what you have written about building A.
- Marti: 5, 10, 15, 20, 25, 50.
- Teacher: Look at those numbers. What's going on?
- John: That's the same as counting by fives. When I got 25, I doubled it to get 50.
- Teacher: How come you got started counting by fives?
- Bob: Because A is five cubes.
- Nate: How come they don't build any buildings like that?
- Teacher: I don't know.
- Monica: I know the numbers in some of the others.
- Teacher: I want to hear more about A.
- Sherry: You can find 25 + 25. 20 + 20 is 40, and not forgetting the two 5s, that's 10. Add 10 + 40 = 50.
- Teacher: You wanted to share how to add 25 and 25. Now let's hear about B. Look at the numbers for Building B.
- Mikal: I discovered something about the fives. The second number goes 5, 0, 5, 0.
- Teacher: Mikal noticed a pattern.
- Marti: On the 10 stories, all of them have zeros at the end.
- Walter: Building B counts by four; 4, 8, 12, 16, 20, and then you double it to get 40.
- Teacher: How is that counting by fours? Can you explain that? [Walter looks up, thinks, and shakes his head to indicate a negative response.] Is that something that's so clear in your head, that you can't figure out how to explain it?
- Walter: Yeah.

- Monica: How I got the numbers. I didn't know exactly how to count by fours, so I built it and then I counted.
- Teacher: Counting by fours doesn't come as quickly as counting by fives, so you found a good way to get the answer.
- Alan: I noticed something. B and C are the same [in the table].
- Teacher: Read us the numbers.
- Alan: 4, 8, 12, 16, 20.
- Marti: B and C have the same amount of cubes, but they're a different shape.
- Tina: So if C was really long and wide, it wouldn't fit. Or if B was small and really tiny, it wouldn't fit.

The teacher asks Tina if what she is saying is that they can be different shapes on the bottom but have the same number of cubes. Tina shakes her head to indicate a negative response. The teacher asks students to make Buildings B and C each three layers high (see FIG. 25.3).

Teacher: How are these [the buildings] the same and different?

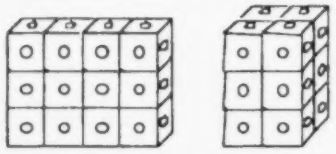


FIG. 25.3.

- Tina: One building goes up in a different way.
- Megan: One building has two on each one and the other has four on each one.
- Bob: One is fatter.
- Monica: I noticed that one building has three holes on one side and six holes on the other.

The teacher asks if she can call the holes windows, and the students agree. The teacher counts 12 windows on one face of Building C and 6 on one face of Building B.

- Nate: That would be a cube if you take off one floor. [The teacher agrees.]
- Mikal: If you build A up to ten, the front would be a building, but what would be the back?
- Teacher: Are you noticing that in one building they just have one room?
- Mikal: Yes.
- Teacher: How many windows would you have in each building?
- Mikal counts 24 in one and 30 in another.
- Teacher: If windows were very expensive, which kind of building would you build?
- Students: A.
- Teacher: Maybe that's why they don't build many buildings that way.

A Rudderless Ship

This episode is an example of a rather diffuse type of "inquiry teaching." The teacher, her aide, and an observer from the curriculum development staff all expressed praise for what happened in class on the day of the discussion. They saw enthusiastic students actively engaged in mathematical exploration, thinking, and communication. We agree. However, we also see substantial difficulties. The discussion resembled a "show-and-tell" sharing session and lacked a clear conceptual focus. It is true that the comments of several students indicate that some nice thinking had occurred. But, that thinking rarely became the focus of discussion. How did the students know that doubling the number of rooms for 5 stories gave the number of rooms for 10 stories? When a student commented that taking a layer off one of the buildings would make a cubical building, there was no follow-up. The teacher pursued the "windows" idea, but the resulting discussion did not seem to lead to anything worthwhile, and it was unrelated to the goals of the lesson. In fact, there was only one attempt by the teacher to get the students to think about one of the major objectives of the lesson from the curriculum designer's point of view—the meaningful application of skip-counting. But this attempt failed to promote the desired discussion.

Furthermore, we wonder how meaningful the students' enumeration of cubes was. Were students determining the number of cubes in each building by spatially organizing the cubes

then enumerating them, or did they merely detect a number pattern that they connected superficially to the structure of the buildings? Subsequently we asked three second-graders who had correctly completed Buildings A through D (only one student had time to complete Building E) to enumerate cubes in a building that measured three by three by four cubes.[2] Two of the three gave incorrect answers. The first counted the cubes one-by-one by columns but omitted the middle four by three layer of cubes; she got 24. When asked to check her answer, she said that there were three layers, but she still enumerated by columns of four. The second student gave an answer of 40, saying, "That's how many it looks like." The third student correctly skip-counted cubes by columns, not by layers. Thus, the students' work on the *Build the Buildings* activity did not seem to have caused any of them to abstract a layer structuring for enumerating cubes in spite of the fact that the activity directed students to determine the number of cubes layer by layer. Instead, it seems that the students used number patterns that were not well connected to spatial quantities. In fact, research shows that older elementary, and even middle school, students have great difficulty meaningfully enumerating cubes in three-dimensional rectangular arrays (Battista & D. H. Clements, 1996).

In conclusion, the original, intuitively derived claim that students' work with the *Build the Buildings* activity would be "good experience" for dealing with cube buildings in later grades is a conjecture that should not have been taken for granted but should have been investigated explicitly. The practical goal of giving students experience with skip-counting in the context of cube buildings seemed to have caused the curriculum developers to ignore relevant research on students' understanding of three-dimensional arrays of cubes.

This episode, then, illustrates what happens when instruction is designed with only general goals in mind and with little or no connection to relevant research on students' cognitive constructions. The educators in the classroom judged the unit successful because it supported a spirit of inquiry in which students were employing skip-counting and making

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[2] These students were tested at a site different from that in which the class discussion took place.

discoveries.[3] However, because the curriculum designers did not adequately attend to the nature of students' mental constructions, as our interviews suggest, it is quite likely that the students' solutions to the instructional problems were the result of superficial application of number patterns. Furthermore, as the classroom observation illustrates, the focus of the lesson was so diffuse that it is unlikely that the activities promoted meaningful learning of a planned set of concepts. Thus, although the unit may have possessed many characteristics of effective mathematics instruction, its lack of conceptual coherence caused it to be much less effective than it could have been.

CURRICULUM DEVELOPMENT AS SCIENTIFIC ACTIVITY

In the mainstream of our culture, scientific knowledge is particularly valued because of the reliability it provides us in dealing with our experiences while in pursuit of our goals. "Our faith [in science] rests entirely on the certainty of producing or seeing again a certain phenomenon by means of certain well defined acts" (Valery, 1957, as quoted by von Glasersfeld, 1995, p. 117). In fact, scientific knowledge is deemed more reliable than commonsense knowledge because it employs the "scientific method" to ensure that it is built up in a way that is explicit and repeatable (von Glasersfeld, 1995). Such knowledge is constructed according to standards that are upheld by scientific communities of scholars committed to particular sets of questions, lines of reasoning, methods for verifying predictions, and shared ways of "seeing" the world (Romberg, 1992a). Members of these

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[3] We do not mean to ignore the benefits of such an inquiry environment on students' beliefs about the nature of mathematics and the way that proper beliefs can affect students' practice of mathematics. But we believe that such inquiry can and should take place while carefully promoting the construction of the specific concepts that students need to acquire to be mathematically literate.

communities review, critique, and build on each other's work to produce knowledge that they take as established and reliable.[4]

To provide explicit and repeatable treatments of phenomena in need of explanation, the scientific method generally includes four steps, all of which must be documented in the dissemination of results (von Glasersfeld, 1995):

1. The conditions under which a phenomenon is observed are made explicit so that the observation can be repeated.
2. A theory is proposed to explain how the observed phenomenon arises.
3. The theory is used to deduce a prediction about an event that has not yet been observed.
4. An attempt is made to produce the predicted event by generating the conditions under which the theory predicted it would happen. The conditions actually generated and the resulting observations are made explicit.

In implementing the scientific method, the social sciences generally place less emphasis on establishing "predictability," focusing instead on finding "plausible interpretations" (Bruner, 1990). Nevertheless, there is still an assumption that the interpretations and conclusions drawn from observations in one context will be generalizable or applicable to similar contexts not yet studied (Martin & Sugarman, 1993). Moreover, social scientists adhere to the fundamental scientific principles of documentation, explicitness, theory building, peer review, and commitment to the systematic cooperative construction of knowledge in their respective disciplines.

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[4] However, the extent to which individual members of scientific communities believe in this knowledge depends on the degree to which they have found that the knowledge personally viable (i.e., it has "worked" for them in the past), and that their understanding of the knowledge is shared by others (von Glasersfeld, 1995).

In mathematics education, research about the learning and teaching of mathematics is one example of scientific activity. It is disciplined inquiry involving the use of procedures accepted by a community of researchers to systematically investigate and explain phenomena in teaching and learning mathematics (Kilpatrick, 1992; Romberg, 1992a). We contend that for curriculum development in mathematics education to progress beyond its current state of chaos and inefficiency, it too must become scientific. To move beyond the "intellectually soft, intuitive, informal and cookbooky" nature historically attributed to "design sciences," such disciplines must develop and adhere to "a body of intellectually tough, analytic, partly formalizable, partly empirical, teachable doctrine about the design process" (H. A. Simon, 1970, as quoted in Wittmann, 1995, p. 362). In fact, it has been argued that only by becoming scientific can such enterprises make "progress" (Kuhn, 1962/1970).

In the following two sections, we provide a framework for making curriculum development more scientific. This framework not only can guide curriculum development efforts, it can serve as a backdrop, or set of standards, against which the process of curriculum development can be studied and evaluated.

ELEMENTS OF A SCIENTIFIC APPROACH TO CURRICULUM DEVELOPMENT

> The whole of science is nothing more than a refinement of everyday thinking.—Albert Einstein (quoted by Donadio, J. Smith, Mesner, & Davison, 1992, p. 370)

To take the first step toward becoming scientific, curriculum developers need to explicate their theoretical standpoints, judgments, purposes, and procedures so that the development process is recorded, shared, and opened to critical reflection and discussion. Such explication includes the six activities described as follows (cf. Gravemeijer, 1994; Romberg, 1992a). Partial implementations of these activities within actual curriculum development projects are

illustrated in Battista and D. H. Clements (1988, in press), D. H. Clements, Battista, and Sarama (in press), and Gravemeijer (1994).[5]

Describe the Problem That the Curriculum Development Effort Seeks to Solve

The "problem" is the creation of a certain type of curriculum. What is the nature of the curriculum? What are the general philosophy and goals for the curriculum; why are they desirable; are they compatible; why should we believe they are attainable? Why is the new curriculum needed? What is novel about the curriculum and what purpose does the novelty serve? Of course, the goals might be quite eclectic, including notions such as teaching mathematics in a way that is consistent with the National Council of Teachers of Mathematics' *Standards* (1989), teaching mathematics through inquiry or problem solving, teaching mathematics in a way that is consistent with the radical constructivist view of learning, offering students meaningful mathematical problems; emphasizing depth rather than exposure to topics, expanding the pool of mathematically literate students, and increasing the success rate for at-risk students. The goals for the curriculum may also extend to factors related to the institutional, economic, political, and cultural contexts in which schools are situated (Apple, 1992).

Provide a Theoretical Explanation of How the Curriculum Development Effort Should Solve the Problem

Based on their beliefs about what mathematics is, how it is learned, and how it should be taught, curriculum developers should have a theory about how their curriculum ensures that students will learn what is intended (Gravemeijer, 1994). The developers' sense making in this regard must be explicit. They must clearly elaborate not only how the development process will lead to a curriculum that will meet the goals they have articulated, but exactly how this curriculum will enable students to learn particular concepts, procedures, and ways of

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[5]We do not think that any curriculum developers, including us, perform all of these activities. But that is not a reason for claiming that such activities should not or cannot be done.

reasoning. They must identify the mental operations that underlie the mathematical capabilities their curriculum seeks to develop and describe how their instructional tasks will engender these capabilities.

To be scientific, a curriculum development effort must describe how it is related to the relevant work of others (Romberg, 1992a). This includes comparing development work with existing curricula, and, more important, explicitly connecting the design of instructional activities to theories of learning and teaching and related research. The developers should describe how extant theories and research are applied or connected to the derivation of specific instructional activities. Predictions about what will happen and why should be made. What cognitive changes are expected in students? How will the proposed instructional activities cause these changes? How will cognitive changes alter students' mathematical behaviors? Several elements of research on students' learning of mathematics that are essential in this regard are described in the next section.

Finally, curriculum developers must explicate their decisions about the inclusion of topics in a curriculum. Too often, such decisions are arbitrary or based solely on tradition. Why should a particular topic be included in the curriculum? What evidence is there that students who study this topic will be able to make it meaningful? How will students make sense of it and why will it be significant for them (as opposed to the curriculum developers)? Often, research on students' learning provides insights into the sense that students make of particular mathematical topics. Other times, curriculum developers must rely on personal theories that they will need to confirm or disconfirm.

As an example, once it had been decided that a curriculum we were developing would deal with volume in fifth grade, we set out to determine what aspects of this topic fifth-graders could make sense of. In examining the research relevant to this area, we found suggestions that older students confused volume and surface area. So we decided to have students make boxes out of square grid paper and determine their surface area and volume, hoping that this would help students discriminate the two concepts. However, when we gave our original instructional tasks to students and carefully examined how they thought about them, we found that merely determining how many cubes were needed to fill a box was far more difficult for students than we had imagined. But it was not because they were confusing surface area and volume. The students intended to enumerate cubes, but because of inadequate spatial

structuring of the three-dimenional array, they were unable to do so correctly (Battista & D. H. Clements, 1996). Furthermore, because of the complexity of the situation, some students apparently failed to maintain the distinction between what they were looking at as they counted (squares) and what they intended to count (cubes); (Battista & D. H. Clements, 1996). Asking such students to determine surface area would have required them to use these squares for another purpose, something that we judged would add to their confusion. Thus, our careful work with students suggested that we abandon our original instructional strategy.

Describe the Testing and Data Collection Procedures

Curriculum developers should explain the specific procedures that will be used for:

- Testing and revising materials.
- Collecting data during the formative period of testing and revising materials.
- Collecting data during the summative evaluation of the curriculum.

Developers must describe the general testing and revising cycle that will be used in designing specific instructional activities. For example, after examining both the research and instructional literature relevant to a particular topic, potential instructional activities will be created for testing. How will those activities be tested? With members of the development team working first with individual students, then, after appropriate revisions, with teachers using materials in their classrooms? What kinds of data will be collected during this formative period? What criteria will be used to decide if the activities are good enough to include in the finished curriculum?

Both for formative and summative evaluation, the curriculum development team must determine what types of data need to be collected to investigate how instructional activities are affecting learning. Qualitative methods seem especially well suited for such investigations (Gravemeijer, 1994; Wittmann, 1995). What combination of teaching experiments, classroom observations, interviews, case studies, and paper-and-pencil tests are needed to make sense of what is going on in the classroom against the background of the theoretical analysis that formed the basis for development? For example, once key cognitive processes, instructional tasks, and categories of students' reasoning have been established using teaching experiments and interviews, it is important to conduct long-term case studies of students' classroom learning and compare the results to those predicted by the underlying theoretical framework.

Document the Development of Instructional Activities and the Collection of Data

Curriculum developers should describe how the process of development proceeds for each major topic in the curriculum. What substantial revisions of materials are made and why? How are these revisions related to data collected about students' learning? All data that are collected on students' learning with newly produced instructional materials should be reported. In fact, by describing things that did and did not work and why, curriculum developers can add to the knowledge base on learning and instructional design instead of attempting to convince others that their curriculum works. Taking this approach necessitates that all testing of instructional tasks be recorded on video- or audiotapes so that the evolution of students' learning with successively better tasks is recorded and can be analyzed. It represents a departure from the standard practice of working on activities until they "seem right," then moving on to an evaluation phase in which students' learning is evaluated formally. If the curriculum development has been connected properly to theory and research on learning, the data collected during this period can form a significant component of an associated research program.

Analyze the Data and Records of Development

The development team should conduct an analysis to determine what has been learned about the effectiveness of the curriculum in promoting students' learning, the curriculum development process, and the theoretical and research base on which the curriculum development was based.

Disseminate the Data and Analysis

Members of the development team should transmit their written analyses to others by publishing them in professional journals and by giving presentations at professional meetings.

THE INTERPLAY BETWEEN CURRICULUM DEVELOPMENT AND RESEARCH

We emphasize that we are not calling for curriculum development to become "research." The scientific development of curricula differs from what we classically think of as research in that

curriculum development focuses on the production of instructional activities, with most of the knowledge gained remaining implicit (Gravemeijer, 1994). In research, knowledge gain is the main concern; the focus is on building theories and descriptions that can be put before the research community.

Making curriculum development scientific will cause the knowledge generated therein to become explicit so that future development efforts can build on it. It will enable mathematics educators to integrate curriculum development and research on learning into a synergistic system, improving not only curriculum development and research but also teaching. To make curriculum development scientific and integrated with research, however, requires that curriculum developers explicitly connect design efforts with research findings, perspectives, and methodologies.

Connecting Curriculum Development to Research Findings

Scientifically sound curriculum development cannot be based solely on *general* theories or philosophies. For example, as we saw in the illustrative example earlier, broad goals by themselves, even if worthwhile, are insufficient to effectively guide the design of instructional activities. A scientific approach to curriculum development requires that *all* relevant research on learning and teaching be considered. There are several findings in that research that mathematics curriculum developers cannot ignore.

First, it is well accepted among researchers that learners construct their own personal knowledge of mathematics rather than receive it ready-made from a teacher or textbook (Hiebert & T. P. Carpenter, 1992; Lester, 1994; Romberg, 1992a; Steffe & Kieren, 1994). For instructional activities to provide opportunities for students to produce such personally meaningful mathematical knowledge, the activities must be "grounded in detailed analyses of children's mathematical experiences and the processes by which they construct mathematical knowledge" (Cobb et al., 1990, p. 130). Curricula that focus on students' behaviors rather than on their cognitions and the nature of their experiences are inconsistent with this premise.

Second, all current research paradigms in mathematics education take it as axiomatic that instructional decisions must be based on knowledge about how students learn particular content, not merely on general theories of learning (Baroody & Ginsburg, 1990; Battista & Larson, 1994; Hiebert & Wearne, 1991; Koehler & Grouws, 1992). In fact, contemporary researchers in mathematics education have gone beyond Piaget's original psychological and

epistemological theories to develop specific models of students' ways of operating as they construct increasingly sophisticated mathematical knowledge *in particular mathematical situations* (Steffe & Kieren, 1994). It is this elaboration and particularization of Piaget's general theory that "makes it possible to consider children's construction of mathematical knowledge in a way relevant to instructional issues" (Cobb et al., 1990, p. 126). Of utmost importance in this view of learning is the identification and description of key cognitive processes that underlie students' competence in various mathematical domains (Hiebert & Wearne, 1991). Thus, to formulate possible learning paths along which students' conceptual development in a particular mathematical domain might progress, curriculum developers must possess a model of how students' cognitive processes and structures in that domain can be gradually transformed into increasingly sophisticated conceptualizations (von Glasersfeld, 1987). Ideas for instructional activities must constantly be evaluated in terms of this model. That is, curriculum developers must frame the goals and success of instructional activities in terms of the cognitive processes these activities are meant to engender.

Third and finally, as reflective practitioners, teachers' knowledge, theories, and belief systems about teaching and learning mathematics influence their instructional plans, decisions, and actions as well as their utilization of curricula (T. P. Carpenter & Fennema, 1991). So if developers expect their curricula to be used as intended, in addition to instructional activities, they must provide materials that help teachers clearly understand not only their approach to learning and teaching, but the various learning paths that teachers are likely to see in their students.

Connecting Curriculum Development to Research Perspectives

There is general agreement among researchers that students' learning of mathematics needs to be viewed from several different complementary perspectives, the most important of which are the psychological and sociocultural (Cobb, 1990; Cobb & Yackel, 1995). From the psychological perspective, we attempt to describe and interpret students' mathematical experiences; we try to understand the nature of students' personal mathematical creations. How do students view situations that we consider as requiring mathematical thinking? What strategies do they use to solve problems and why? What mental operations give rise to the mathematical concepts, strategies, and experiences that students are inferred to have? These include general operations such as abstraction, assimilation, and accommodation (von

Glasersfeld, 1995), and those utilized in specific situations such as the unitizing and partitioning operations described by Steffe and Cobb (1988), or the structuring, coordinating, and integrating operations described by Battista and D. H. Clements (1996).

From the sociocultural perspective, we attempt to describe the social and cultural factors that affect students' construction of mathematical meanings. We try to describe the development and functioning of mathematical classroom cultures. How are classroom norms and practices in mathematics formed? How are students' beliefs about their own and the teacher's roles in the classroom, as well as their beliefs about the mathematical enterprise, affected by their participation in classroom cultures? How does socially supported thought—say, the spirit of inquiry—become a mathematics classroom norm, and how does it become "internalized" by students? Though we generally focus our analysis on the culture of the classroom, sometimes we must extend it to the culture of the surrounding school and community (Cobb & Yackel, 1995).

Significantly, the relative weights that developers give to these two perspectives might greatly affect the procedures used in developing a curriculum. For instance, as developers design instructional tasks, how do they envision the resulting materials being used? If they intend to have students construct and sufficiently refine ideas without interacting with others, they will incorporate opportunities for perturbation and self-checking of students' theories into the tasks. They will develop and test materials first by conducting teaching experiments with individual students. Psychological factors will be at the forefront of their theoretical perspective. It is only during the second phase of development, when they have teachers test the materials with whole classes of students, that they will attend to sociocultural factors.

Alternatively, developers might envision instructional tasks as tools for guiding the evolution of mathematical practices and discourse in the classroom culture. A critical component in the design of these tasks is the expectation that students' learning will result from their adoption of socially negotiated classroom practices developed in response to the tasks. Initially, these developers might envision and test activities with whole classes of students; their perspective is primarily sociocultural. In our view, placing initial emphasis on sociocultural factors can be valid. But it must be accompanied by the psychological perspective—the developers must also carefully examine individual students' learning.

We conclude, therefore, that both the psychological and the sociocultural perspectives are critical in a scientific approach to developing mathematics curricula. Taking a psychological

perspective is essential because the goal of instructional tasks is to change the ways that students think; we want to induce students' development of increasingly more sophisticated cognitive operations for dealing with particular domains of mathematics. But because students will be using these tasks within a classroom culture consisting of many students and a teacher, we also need to take a sociocultural perspective to understand how students' individual construction of meanings is related to their participation in the classroom community. Though it might be useful for one perspective to take precedence in the initial design of an instructional activity, it is imperative to consider both in the overall conceptualization and development of the activity.

A good example of how these two perspectives can be utilized in curriculum analysis is offered by the *Build the Buildings* activity described earlier. The educators in the classroom seemed to take primarily an implicit sociocultural perspective, judging that members of the classroom culture were productively engaged in mathematical inquiry.[6] But it was the psychological perspective that revealed the shortcomings in the instructional unit. The unit did not seem to effectively encourage individual students' construction of a set of targeted concepts. We could return to a sociocultural analysis to relate individual learning to the operation of the classroom culture. In a productive culture of inquiry, discussions involve more than show and tell; students listen to and critique each others' comments so that knowledge is acquired collaboratively.

Connecting Curriculum Development to Research Methodologies

Methodologies utilized in research on mathematics learning frequently can be adapted to provide scientific rigor in curriculum development. As an example, we illustrate how teaching experiment-based methodologies advocated by constructivists (Steffe, 1996; von Glasersfeld, 1987b) can be used in the scientific development of mathematics curricula.

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[6] It is likely that much of what the students did in this regard was due to the mathematical culture the teacher had engendered over many months.

Preliminary Work. Curriculum developers should determine the categories of mathematical minds, or ways of operating, that students have with respect to particular mathematical topics that are to be taught. In addition to reviewing relevant research, this should be accomplished by interviewing students or engaging them in exploratory teaching. In this step, curriculum developers generate tentative theoretically grounded models of students' present conceptions of and ways of reasoning about target topics. They also develop hypotheses about how students' current ways of operating can be guided toward adult competence.

Teaching, Model Building, and Hypothesis Testing for Individual Students. During teaching experiments conducted with individual (or perhaps pairs of) students, curriculum developers should formulate and test detailed working models of how students construct specific areas of mathematical knowledge. At this time, the goals should be to gain understanding of students' mathematical sense making and ways of operating, and to experiment with methods of modifying student's understandings and ways of operating. The psychological perspective will be dominant. By observing students' work on appropriately selected tasks, curriculum developers formulate hypotheses concerning what students' in each category of mathematical mind might learn. To test these hypotheses, developers present tasks to students and allow them to establish their own history of constructions.

Retrospective Analysis for Individual Students. After collecting data on individual students, curriculum developers should use videotape records of teaching episodes to perform a retrospective analysis of students' actions and interactions during the experiment. As part of this analysis, developers should constantly attempt to determine exactly what mathematical ideas students were personally constructing as they worked on instructional tasks, no matter how much these ideas differ from standard mathematical concepts.

Teaching, Model Building, and Hypothesis Testing with Classes of Students. Once curriculum developers have sufficient knowledge to effectively teach the desired topics interactively to individuals, they should use this knowledge to design instructional sequences that a teacher might use for an entire class. These sequences must accommodate the different mathematical minds that will exist in a class of students. Developers should test these instructional sequences and the models underlying them by conducting classroom teaching experiments in which whole classes are taught by students' regular classroom teachers. At this

time, the goals should be to ascertain how students construct knowledge of the target concepts within a classroom setting, and to determine ways of modifying instructional sequences to maximize students' learning as they interact with the materials, the teacher, and other students. Although the sociocultural perspective will dominate, the psychological viewpoint cannot be forgotten. One way to accomplish the synthesis in perspectives required in this step is to conduct case studies of individual or pairs of students within the context of the study of the whole class.

Retrospective Analysis for Classes of Students. Curriculum developers should construct a retrospective analysis of classroom teaching experiments. In this analysis, they should seek to understand how students with different mathematical minds learned within classroom situations, and how aspects of classroom cultures affected individual student learning.

Final Curriculum Revision and Scientific Model Building. Curriculum developers should make final revisions to the curriculum, which includes instructional tasks, rationale for tasks, descriptions of how to implement tasks, and conceptual analyses of how students are likely to progress on them. They should also conduct a research-based analysis of learning and instruction within the completed curriculum. In this analysis, the activity of model building that was present throughout the development of instructional activities is revisited with the intention of building a final model that can be presented to the research community. Explanations in this model should be based on concepts in the core of the research program like assimilation, accommodation, mental model, scheme, reflection, and forms of social interaction, as well as concepts particular to the topic at hand, such as unitizing, structuring, coordination, and integration. Together, the retrospective analysis and scientific model building should provide a complete account of the development of students' mathematical knowledge and the effects of instruction on student learning within the completed curriculum. This account should include a description of how particular instructional tasks, learning environments, and sociocultural factors affected students' learning.

Benefits

In addition to adding rigor to the curriculum development process, employing appropriately adapted research methodology can make the acquisition of knowledge during curriculum development methodologically sound and properly framed theoretically. Consequently, the resulting knowledge about learning and teaching, which traditionally has remained implicit,

can meet the standards required to be published in research journals, adding vital information about learning and teaching to our knowledge base. Moreover, such "curriculum development research" can help practitioners better appreciate mathematics education research as applicable to classroom use.

Another benefit of applying research methodology to curriculum development is that it can focus attention on traditionally neglected aspects of the development process. For instance, both Guba and Lincoln (1989) and Cobb (1996) suggested that researchers include the perspectives of major stakeholders in research studies. Implementing this suggestion would require curriculum developers to systematically collect and represent the views of teachers and students who are using the newly developed instructional materials. Instead of researchers conducting studies about teachers' use or misuse of innovative curricula, or curriculum developers merely taking informal note of teachers' views—both of which are valuable activities—developers should systematically acquire teachers' comments about using the materials. They should then honestly represent the teachers' views in scholarly reports on the curriculum. Similarly, developers could gather and present the views of students.

POSSIBLE CRITICISMS OF SCIENTIFIC CURRICULUM DEVELOPMENT

There are several potential criticisms that could be directed toward a scientific approach to curriculum development. In each case, however, we find that examining the criticism reveals strengths of this approach. First, not every mathematical topic that one wishes to include in a curriculum has been researched sufficiently to provide clear guidance for the design of instructional tasks. Consequently, curriculum designers often will have to conduct their own developmental research on these topics. The advantage of approaching curriculum development scientifically is that work done in this regard enters the professional literature in mathematics education, thus increasing the store of knowledge for curriculum development and research alike.

Another possible criticism of a scientific approach to developing curricula is that it will slow down the pace of development. Actually, we view this as an advantage. Rarely have funding agencies given curriculum development projects enough time to accomplish their

goals. Quality curriculum development cannot be rushed. Much work in this field is like most current mathematics instruction—forever doing the same things, but doing them badly, so they have to be repeated. In fact, construing curriculum development as a scientific undertaking should cause the mathematics education community to drastically rethink the time frames and resources required for this essential activity.

It also might be argued that a scientific approach will stifle the creativity of developers. Though having standards for curriculum development might feel constraining to some initially, developers will learn to channel their creativity within these constraints, as have practitioners in other scientific disciplines. In fact, they will be members of the community that cooperatively establishes these constraints.[7]

An additional potential criticism to making curriculum development scientific is that it will make some curriculum development efforts legitimate and others illegitimate. Although some may view this practice as exclusionary, it is a necessary price for making curriculum development scientific. Indeed, the practice of any science is restricted to those who are willing to learn its core knowledge and become fluent in its established ways of operating. Besides, in today's haphazard world of curriculum development, there are many efforts that should be excluded (Wittmann, 1995). For instance, many current commercial projects are doing more harm than good. Too often, these projects tout traditional curricula with surface-level changes

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[7]Thinking of scientific activity as problem solving can help researchers and scientifically minded curriculum developers avoid putting themselves into methodological straitjackets that impede, not accelerate, progress. When we are perplexed by a difficult problem, we often cycle through different phases in our attempts at finding a solution. In one phase, we try to generate good ideas, potentially fruitful paths to a solution. During this phase, it is unproductive to attend to all the details, to verify each step in possible solution paths. Doing so can waste time, and ultimately prove futile. In the second phase, after we have chosen a solution path, careful analysis is mandatory; we must painstakingly check our proposed solution. Because the science of curriculum development, and some would say research in mathematics education, is still in its infancy, we should be careful not to prematurely overspecify rigid methodologies for its practice.

as "reform curricula," giving school personnel who adopt them the impression that needed changes have occurred. Because the lack of genuine change is obscured, the mathematical miseducation of students continues.

Finally, it might be argued that making curriculum development scientific will devalue the wisdom of practice of experienced designers and teachers. It is true that intuition and experience can be valuable in guiding the development of curricula. But this "craft knowledge" can also be wrong (Leinhardt, 1990). To separate the wheat from the chaff, to refine this knowledge and preserve it so that others can build on it instead of duplicating it, requires a scientific approach. Indeed, taking a scientific approach is probably the only way to deal with the difficult problem of recording and learning from the "wisdom of practice" of experienced curriculum developers (cf. Leinhardt, 1990).

CONCLUSION

Curriculum development, teaching, and learning are the core activities in the educational enterprise—educators develop curricula and teach so that students learn. Through research in mathematics education, the processes of teaching and learning mathematics have been brought under the scrutiny of scientific analysis, resulting in clear knowledge gains about both. But, so far, the process of curriculum development has eluded the organization, analysis, and progress that can come from adopting appropriate scientific standards. Until we make curriculum development more scientific, firmly establishing this critical link between research and practice, mathematics teaching will continue to flounder.

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